№1 4 шара, 3 ячейки, ξ – число пустых

$$\mathbb{P}\{\xi = 0\} = \frac{C_3^1 \cdot C_4^2 \cdot C_2^1}{3^4} = \frac{3 \cdot 6 \cdot 2}{3^4} = \frac{36}{81}$$

$$\mathbb{P}\{\xi = 1\} = \frac{C_3^1\left(C_4^2 + C_4^3 \cdot C_2^1\right)}{3^4} = \frac{42}{81}$$

$$\mathbb{P}\{\xi = 2\} = \frac{C_3^1}{3^4} = \frac{3}{81}$$

ξ:

0	1	2
$\frac{4}{9}$	$\frac{14}{27}$	$\frac{1}{27}$

№2

(ξ, η)	η		
ξ	-1	0	1
0	1/4	3/8	1/8
1	1/12	1/8	1/24

ξ:

0	1
$\frac{3}{4}$	$\frac{1}{4}$

$$\zeta = \xi^2 \eta$$

ζ:

-1	0	1
$\frac{1}{12}$	$\frac{7}{8}$	$\frac{1}{24}$

η:

-1	0	1
$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

$$\operatorname{cov}(\xi, \eta) = E\xi\eta - E\xi E\eta = E\zeta - E\xi E\eta =$$

$$= -\frac{1}{12} + \frac{1}{24} - \frac{1}{4}\left(\frac{1}{6} - \frac{1}{3}\right) = 0$$

• №3

$$p_\xi(x) = \begin{cases} cx^3, & 0 \leq x \leq 2 \\ 0, & x \notin [0,2] \end{cases}$$

$$\int_0^2 cx^3 dx = \frac{cx^4}{4}\Big|_0^2 = \frac{16c}{4} = 4c = 1 \Rightarrow c = \frac{1}{4}$$

$$p_\xi(x) = \begin{cases} \frac{x^3}{4}, & 0 \leq x \leq 2 \\ 0, & x \notin [0,2] \end{cases}$$

1) $x \in [0,2]$

$$F_\xi(x) = \int_0^x \frac{t^3}{4} dt = \frac{x^4}{16}$$

$$F_\xi(x): \begin{cases} 0, & x < 0 \\ \frac{x^4}{16}, & x \in [0,2] \\ 1, & x > 2 \end{cases}$$

$$\mathbb{P}\{1 \leq \xi \leq 1.5\} = F_\xi(1.5) - F_\xi(1) = \frac{\frac{3^4}{16}}{16} - \frac{1}{16} = \frac{65}{256}$$

$\eta = (\xi - 1)^2$

$$F_\eta(t) = \mathbb{P}\{(\xi-1)^2 < t\} = \mathbb{P}\{-\sqrt{t} < \xi - 1 < t\} =$$

$$= \mathbb{P}\{1 - \sqrt{t} < \xi < 1 + \sqrt{t}\} = F_\xi(1+\sqrt{t}) - F_\xi(1-\sqrt{t}) +$$

$$F_y(t)=\begin{cases}\dfrac{(1+\sqrt{t})^4-(1-\sqrt{t})^4}{16}+1, & 0\leq t\leq 1\\ 1, & t>1\\ 0, & t<0\end{cases}$$

$$P_y(t)=\begin{cases}\dfrac{1+3t}{4\sqrt{t}}, & 0\leq t\leq 1\\ 0, & t\notin[0,1]\end{cases}$$

$$Ey=\int_0^1\frac{(1+3t)t}{4\sqrt{t}}dt=\int_0^1\frac{t}{4\sqrt{t}}dt+\int_0^1\frac{3t^2}{4\sqrt{t}}dt=$$

$$=\frac{1}{4}\int_0^1 t^{1/2}dt+\frac{3}{4}\int_0^1 t^{3/2}dt=\frac{1}{4}\cdot\frac{2}{3}t^{3/2}\Big|_0^1+$$

$$+\frac{3}{4}\cdot\frac{2}{5}\cdot t^{5/2}\Big|_0^1=$$

$$=\frac{1}{6}+\frac{3}{10}=\frac{7}{15}$$

$$Ey^2=\int_0^1\frac{(1+3t)t^2}{4\sqrt{t}}dt=\int_0^1\frac{t^2}{4\sqrt{t}}dt+\int_0^1\frac{3t^3}{4\sqrt{t}}dt=$$

$$=\frac{1}{4}\int_0^1 t^{3/2}+\frac{3}{4}\int_0^1 t^{5/2}dt=$$

$$= \frac{1}{4} \cdot \frac{2}{5} + \frac{3}{4} \cdot \frac{2}{7} = \frac{1}{10} + \frac{3}{14} = \frac{11}{35}$$

$$Dy = \frac{11}{35} - \frac{49}{225} = \frac{152}{1575}$$

№4

$$p_\xi = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}, \quad x \in (-\infty, +\infty)$$

$$\zeta = \xi^2 + \eta^2$$

$$F_\zeta(t) = \mathbb{P}\{\xi^2 + \eta^2 \leq t\} = \iint\limits_{x^2+y^2 \leq t} \frac{1}{2\pi} e^{-\frac{1}{2}x^2} e^{-\frac{1}{2}y^2} dx dy =$$

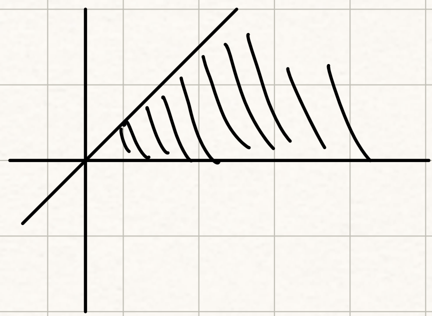
$$= \frac{1}{2\pi} \iint\limits_{x^2+y^2 \leq t} e^{-\frac{1}{2}(x^2+y^2)} dx dy = \frac{1}{2\pi} \int_0^{2\pi} d\varphi \int_0^{\sqrt{t}} r e^{-\frac{1}{2}r^2} dr =$$

$$= \frac{1}{2\pi} \int_0^{2\pi} d\varphi \int_0^{\sqrt{t}} d\left(-e^{-\frac{1}{2}r^2}\right) = \frac{1}{2\pi} \int_0^{2\pi} 1 - e^{-\frac{1}{2}t} d\varphi =$$

$= 1 - e^{-\frac{1}{2}t}$ – пок. распр-е

$$E\zeta = 2, \quad D\zeta = 4$$

№5 $p(x,y) = \begin{cases} e^{-x}, & 0 \le y \le x \\ 0, & \text{иначе} \end{cases}$



$$p_\xi(y) = \int_y^{+\infty} e^{-x}\,dx = -e^{-x}\Big|_y^{+\infty} = e^{-y}$$

$$p_\xi(x) = \int_0^x e^{-x}\,dy = ye^{-x}\Big|_0^x = xe^{-x}$$

$p_\xi p_\eta = xe^{-x}e^{-y} \neq p_{\xi,\eta} \Rightarrow$ не незав-е